

LAMM MODEL FOR NONPERFORMING LOAN PORTFOLIOS' MARKET VALUE DETERMINATION THROUGH MULTIVARIABLE ESTIMATE

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Abstract

The application of the methodology integrated in the model lets us estimate multiple variables and state a portfolio value including loans with different rates and variables that require a differentiated treatment; this means that the model is applicable to determine, for example, the portfolios' value consisting of consumption loan groups, other loans of other type, supported by mortgage securities and other loans of any type, not supported by any securities and, at the same time, such assets may be rated as matured portfolio loans or portfolio loans in force, and have been submitted to an administrative recovery process or a legal recovery proceeding.

INTRODUCTION

The present model was designed by the Business Intelligence Service of Sayco, for the appraisal of loan assets integrated in portfolio packages showing incompatible variables for the purposes of determining an estimate through a linear process.

The application of the methodology integrated in the model lets us estimate multiple variables and state a portfolio value including loans with different rates and variables that require a differentiated treatment; this means that the model is applicable to determine, for example, the portfolios' value consisting of consumption loan groups, other loans of other type, supported by mortgage securities and other loans of any type, not supported by any securities and, at the same time, such assets may be rated as matured portfolio loans or portfolio loans in force, and have been submitted to an administrative recovery process or a legal recovery proceeding.

Taking into account that in order to determine an expectation of assets' recovery, the portfolio subject matter of the model design consists of three types of loans: (a) matured portfolio loans supported by real estate securities; (b) matured portfolio loans not supported by real estate securities; and (c) portfolio loans in force generating foreseeable cash flows; the model consists of two specific appraisal methodologies to be applied to each type of asset above mentioned.

The methodology applied to the ERSNA Model – Sayco (Estimated Return model for Secured Nonperforming Assets) that grounds its design in the estimate of the recovery expectation of the securities supporting the assets and their monetization was incorporated for the

matured portfolio loans supported by real estate securities.

The methodology applied to the ERCON Model – Sayco (Estimated Return Model for Consumer Nonperforming Loans) that grounds its design on the estimate of the recovery expectation of a proportion under the total value of the unpaid principal balance (UPB) disclosed by the assets was incorporated for the matured portfolio loans not supported by real estate securities.

A standard methodology to appraise discounted flows to determine the prospective present value of flows generated by this kind of assets was incorporated for the portfolio loans in force generating foreseeable cash flows.

The three incorporated methodologies seek, independently, to determine a net recovery expectation for the assets groups to which these are applied and, as soon as this variable is determined in each of the appraised assets' segments, the values of the three segments are integrated so as to set forth the present value of such expectation through the application of an annual discount rate, common on the three assets segments and to determine a pondered present value for the total balances of the loans constituting the portfolio.

This tool serves to estimate the revenues expectation shown by this kind of assets, with an acceptable degree of certainty in order to have the information necessary to get acquainted with the probable price range prior to the decision making process, either to sell the assets, to admit some bid as the process winner, or otherwise, to declare a process as deserted.

OBJECTIVES OF LAMM MODEL

To have a tool which, from the estimate of the prospective probable cash flow of loan assets consisting of loans of other nature and in a different situation of maturity and treatment for their recovery allows the determination, with an acceptable degree of certainty, of the market value range estimated as reasonable to designate a winner participant in a sale process.

To have a tool which, from the estimate of the prospective probable cash flow of loan assets consisting of loans of other nature and in a different situation of maturity and treatment for their recovery allows the determination, with an acceptable degree of certainty, of the market value range estimated as reasonable to determine the amount of a purchase bid for such assets.

DESIGN

Contrary to the appraisal practiced to productive loan assets (Performing Assets), where the objectives of the models seek for the measurement of the non-compliance (non-performance) loan risk, understood as the potential loss that the counterparty is able to comply with its financial obligations under the conditions stipulated by a contract, granting an estimate value to the expected loss, the status of the loan constituting the portfolios which are subject matter of the applicable of the developed model, has turned to be a predefined critical status for the institution which offers the portfolios for sale, considering as non-productive assets (Nonperforming Loans) in the books of the originating institution, defined in agreement with the methodology discussed at the Conclusion

Memorandum issued by the Work Team of the Intersecretariat on National Accounts of the International Monetary Fund, ascribed to the United Nations Statistics Division.

Then, contrary to the loan risk measuring models, where measures related to nonperformance, estimate of losses and their probability of distribution to determine the expected losses and non-expected losses are applied, and in view of the fact that the loan constituting the portfolios, subject matter of the model application has reached the maximum possible level of loss on the amount of capital invested by these assets originating institution as a result of their exposure to loan risk, for a specific time horizon, the developed model (LAMM) provides estimates for the expected recovery and their distribution of probability.

The purpose of the LAMM model measurement is to estimate this distribution, since through such distribution we are able to determine the expected recovery and the unexpected recovery on the loan portfolio, which is subject matter of the determination of the purchase bid or otherwise, of the portfolio management by the purchasing firm.

The **recovery expectation** of a portfolio represents the amount of revenues produced by the loans constituting such portfolio, as a result of the management effort conducted, for a determined time horizon.

Exposed Amount (EA). This concept represents the maximum amount that may be expected as product from the recovery of assets and is considered as the amount exposed to a management effort for recovery at the end of a period previously stated.

With respect to the loans not supported by real estate securities, the Exposed

Amount is represented by the unpaid principal balance of the assets constituting a determined portfolio.

Regarding those loans supported by real estate securities, the Exposed Amount is represented by the Net Value of Securities supporting them; this means, the value determined, through an authorized appraisal, as the value of security market, less the value of other registered liens prior to the registration that supports the granting of the loans submitted to the process of the model.

Recovery Rate (RR). RR is the percentage represented by the amount of recovery in relation to the exposed amount and derives from the calculation of an estimated discount rate on the exposed amount, which is granted to the holder credited as motivation factor to make the payment of the balance resulting from subtracting such discount from the sum of the exposed amount.

Probability of Payment (PP). PP represents the relative frequency at which are present the events that, upon the expiration of the terms stated for the recovery, the credited has paid the debt in full or, otherwise, the probability that the credited does not make the payment during or at the end of the term stated for the analysis.

The Recovery Expectation may be estimated according to the following:

$$ER = EA \times Rr \times PP \quad (\text{Eq.1})$$

Where:

ER	=	is the expected recovery
EA	=	is the exposed amount
RR	=	is the recovery rate
PP	=	is the probability of payment

Methodology to estimate the variables in the determination of the gross ER.

Exposed Amount (EA). For the purposes of the LAMM Model, the methodology for the EA determination derives from the application of some of the alternative procedures described below, on grouped loans according to the nature of the securities supporting their granting.

This implies that in order to determine the EA, it is necessary to carry out the prior configuration of loan groups from the variable represented by the securities supporting them, based on the following set structure:

Loans Portfolio in Force.- In such cases, the EA is determined by the construction of the prospective revenue flow expected from the unpaid principal balance determined by the originating institution upon the application of the LAMM by means of the following procedure:

$$EA = UPB \cdot (1 + AIR)^{(tm/12)} \quad (\text{Eq.2})$$

Where:

EA	=	is the Exposed Amount
UPB	=	is the Unpaid Principal Balance
AIR	=	is the Annual Interest Rate at which the loan was documented
tm	=	is the total number of months pending of payment

Loans with no security support.- In such cases, the EA is determined by the assets' originating institution, defined according to the methodology discussed at the Conclusion Memorandum with respect to the treatment of matured loan portfolio for macroeconomic statistic purposes

issued by the Work Team of the Intersecretariat on National Accounts of the International Monetary Fund, ascribed to the United Nations Statistics Division (Exhibit 1 to this instrument), that corresponds to the amount of the unpaid principal balance entered at the loans conforming the portfolio; also, it corresponds to the principal's value at which collection rights are granted, through a sale transaction, to the acquiring firm.

Loans with security support.- In such cases, the EA is determined by the difference resulting from discounting the amount of debts entered as liens of the loan security, prior to that of the lien corresponding to the loan which value is to be estimated by means of the LAMM Model application, at the value determined in the loan security appraisal authorized as market value. This EA may be estimated by means of the following procedure:

$$EA = SMV - LI \quad (\text{Eq.3})$$

Where:

EA	=	is the Exposed Amount
SMV	=	is the Security Market Valu
LI	=	is the amount of other Liens registered previously

Recovery Rate (RR).- This variable is calculated from the coefficient resulting from dividing the total amount estimated as recoverable by the amount of the loans exposed, considered as recoverable:

$$RC = \sum RA / \sum EA_r \quad (\text{Eq.4})$$

Where:

RC	=	The recovery coefficient
RA	=	The total recoverable amount during the analysis period
EA _r	=	The total exposed amount registered in the loans that will make payments during the analysis period

The determination of this variable takes place in relation to the type of related assets and derives from the application of some of the alternative procedures described below on grouped loans according to the type of rating of the involved loan agreements.

This implies that, in order to determine the RR, it is necessary to carry out the previous configuration of loan groups from the variable represented by the types of loan agreements, based on the following set structure:

Consumer loans, including current account credits (credit cards, payroll loans, revolving credit) and loans to purchase consumption goods.- In these cases, the RR is determined from extrapolation of the trend of the last pondered recovery coefficients, registered by the sources of the firm for consumption loans in the geographic area and the number of the respective months matured during the 12 previous months.

Mortgage loans to purchase or improve housing.- In these cases, the RR is determined from the extrapolation of the trend of the last pondered recovery coefficients, registered by the sources of the firm for housing mortgage loans within the geographic area and the number of the respective months matured during the 12 previous months; and

Loans granted for commercial and industrial purposes.- In these cases, the RR is determined from the extrapolation of the trend of the last pondered recovery coefficients, registered by the sources of the firm for commercial and industrial loans within the geographic area and the number of the respective months matured during the 12 previous months.

The sources of the firm that provide permanent information on the performance of loan portfolios for the construction of statistic tables allowing the extrapolation of date in the estimate of the RR are confidential, and we may mentioned only in general terms, taking into account among them 5 credit institutions, a loan information institution and 4 secondary market portfolio managing institutions.

Probability of Payment (PP). For the determination of this variable we have selected the application of the Probit model of qualitative election based on regular accumulated distribution function, which also allows to explain the existing relation between a set of variables or attributes and a dichotomic variable that represents the election between two qualitative options denominated as success or failure, which are denoted with the values 0 and 1.

The model estimates the probability that a credited with certain features or attributes elects the option associated to $y = 1$ by means of the following:

If the decision to elect option A depends on a non-observable index I_i that is determined from the attribute vector X_i in such a way that when the I_i value increases, the probability associated to the election will be higher.

This index may be expressed as follows:

$$I_i = \beta X_i + \varepsilon_i \quad (\text{Eq.5})$$

Additionally, it will be assumed that there is a threshold value of the index I_i denominated as I_i^* , so that should I_i exceed the value I_i^* , option A will be elected. Both, the threshold value of the index I_i^* and the value of the index I_i are non-observable; nevertheless, if it is assumed that it has a regular distribution it is possible to estimate the function parameters.

In this model, it is assumed that the index I_i is distributed regularly, therefore, the probability that I_i^* is lower or equal to I_i may be calculated from the regular accumulated distribution function, as follows:

$$P_i = \text{Rp}[Y_i = 1 \mid X_i = \text{Rp } I_i^* < I_i] = F(I_i) = \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{I_i} e^{-\frac{t^2}{2}} dt$$

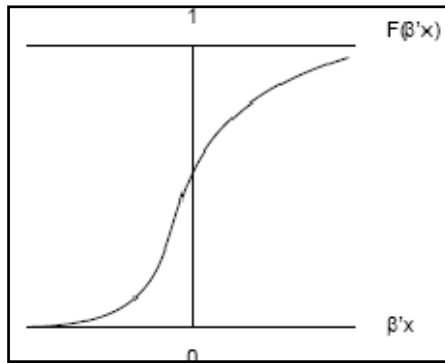
$$\text{Rp}[Y = 1 \mid X_i] = \int_{-\alpha}^{I_i} \phi(t) dt$$

$$\text{Rp}[Y = 1 \mid X_i] = \Phi(\beta x)$$

Where:

ϕ is the standard regular distribution with zero media and variable one

Since P_i represents the probability that the event takes place, P_i is measured as the area under the standard regular curve from $-\alpha$ up to I_i , as shown in the following graph:



Theoretical Basis of the Probit model characteristic as a qualitative election model.

The principle in the applicable of the Probit model is that the probability that an event occurs depends on certain attributes that feature the individual who makes the choice; this means that a credited has the option to elect between two alternatives. Y_i is defined as a dichotomic variable that will take the value of 1 if the individual elects the alternative denominated as to make the payment and 0 in another case.

The model assumes that a credited has a series of attributes that feature him/her, which are denoted as X_i , where the attributes are related to the decision made by the individual. From the previously stated, either if β is the vector of parameters that measures the impact of such variables in relation to the probability to elect the option $Y = 1$, the model that relates these variables is:

$$Y_i = \alpha + \beta x X_i + \varepsilon_i$$

This model expresses the dichotomic variable Y_i as the linear function of the X_i attributes.

The conditional expected value $E[Y_i | X_i]$ is interpreted as the conditional probability that the debtor elects the payment alternative $Y = 1$, given in a set of attributes X_i that characterizes it, that is to say:

$$Rp[Y_i = 1 | X_i] = P_i = E[Y_i | X_i]$$

The previously stated, due to the nature of the dichotomic variable

This way, if the attribute ratio X and the index is positive ($\beta > 0$) then, the index value will be higher, increasing thus the probability that the event $Y = 1$ takes place, since the probability associated to a value above the index will be higher.

Now, in order to obtain information on I_i , as well as of the parameters of the model, we will calculate the inverted of the accumulated function so as to obtain:

$$I_i = F^{-1}(P_i) = \beta X$$

Where:

F^{-1} is the inverse to the accumulated distribution function. Taking into account the previously stated, and in view of the dichotomic nature of the existing dependent variable, as already mentioned above, that:

$$E[Y_i = 1 | X_i] = P_i$$

$$P_i = Rp[Y_i = 1 | X_i] = Rp(I_i^* < I_i) = F(I_i)$$

Finally, the Probit model may be expressed as:

$$I_i = F^{-1}(P_i) = \beta X$$

Y_i	Probability
1	P_i
0	$1-P_i$

$$E[Y_i | X_i] = \beta X$$

If X is the vector of the explicative variables and β is the impact of such variables in relation to the probability, it may be expressed in relation to X , as:

$$Rp[Y = 1] = h(\beta X)$$

$$Rp[Y = 0] = 1 - h(\beta X)$$

$$h(X, \beta) = \beta X$$

Since $E[Y | X] = h(X, \beta)$, the returning model is:

$$Y_i = \beta X_i + \varepsilon_i$$

Probit assures that the estimated probabilities are within the interval (0,1) keeping a non-linear relation with the attribute vector, in an attempt to estimate the vector of parameters βy , and from this, to obtain estimates for the index li and for the probabilities P_i , which will be indicating that for an individual with certain set of attributes, the probability that he/she elects the option A is measured by the value of such probability.

Because of the non-linearity of the model, the procedure used is of maximum probability, guaranteeing the efficiency and consistency properties of the estimators.

The application of a Probit model resolves the limitation presented by the methodology RPA in matter of quantization of the effect of probability determinants of non-performance, in this case, of making payment, since it allows

the identification and quantization of such effects so as to estimate and foreseen it.

Determination of attributes derived in vector X_i

The determination of the attributes that integrate the vector X_i is conducted in accordance with the nature of the administrative process for the recovery of assets that is taking place on the date of the LAMM application and derives from the application of some of the alternative procedures described next, on the loans grouped according to the type of recovery process to which assets are submitted.

This implies that in order to determine the assignable attributes so as to integrate the X_i vector, the configuration previous to the loan groups from the variable represented by the administrative processes for their recovery to which they are being submitted at the time of the LAMM application must be carried out, based on the following set structure:

a) Loan portfolio in force, which are being submitted to a management follow-up process or which are not being subject to any recovery process.- In these cases, no attributes are determined for the integral vector X_i , since the variable y is granted a direct value of $y = 1$;

b) Matured portfolio loans being submitted to a motivation administrative procedure in order to make the payment or which are not under any recovery process.- In these cases, the attributes corresponding to the vector X_i is determined taking into consideration the following probabilistic factors: X_i = Geographic location of the credited and proportion of portfolio in force that derives in matured portfolio rated B to E in agreement with the circular letter B10 from the CNVB (NSEC), submitted to an administrative process of

recovery in such geographic area; X_2 = Type of loan and proportion of portfolio in force that derives in matured portfolio under a recovery administrative process in such type of loan; X_3 = Amount paid in relation to the total granted up to the date of the LAMM application; X_4 = Data available to be able to locate the credited; X_5 = Existence and type of guarantors or additional securities granted; X_6 = Range of delinquency in number of months; X_7 = Existence and quality of the documentation covering the granting of the loan; X_8 = Other incidental factor that may be relevant in each case. In this case, the vector X_i is determined by $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8$.

c) Loan in matured portfolio under recovery legal proceeding.- In these cases, the attributes corresponding to vector X_i are determined taking into consideration the following probabilistic factors: X_1 = Geographic location of the loan and characteristics of the applicable legislation of the involved entity in relation to the probability of an actual trial; X_2 = legal strength of the loan granting; X_3 = Type of trial and procedural stage of the relevant lawsuit; X_4 = Soundness of legal proceeding determined based on a legal opinion elaborated by the holder counselor of the proceeding; X_5 = Other incidental factors that may be deemed relevant in each case. In this case, the vector X_i is determined by $X_1 + X_2 + X_3 + X_4 + X_5$.

Estimate of variable for the determination of the net value of ER.

As soon as the gross value of ER is known through the application of the procedure described in the equation 1 (Re: Eq. 1), we are seeking to determine the net value of ER by means of the application of the following equation:

$$ER' = ER - TGC$$

Where:

ER'	=	ER net value
ER	=	Gross value of the expected recovery
TGC	=	Transformation global costs of assets in liquidity resources

In this equation, the factor TGC consists of the sum of the costs implied in: (a) the acquisition of assets, not taking into account the funding and the funding costs; (b) the reception, organization, preparation and administration of assets; (c) the recovery of loans, including legal and non-legal expenses and fees; and (d) all other costs involved in the recovery, transfer of ownership and monetization of the securities, as soon as these would have been recovered.

Determination of the ER' present value for the purpose to estimate the reference minimum price in a portfolio sale process

The ER' present value is determined by applying the following equation:

$$RP = ER' / (1 + DR)^{(tm/12)}$$

Where:

RP	=	Reference price for the appraised portfolio.
ER	=	ER net value.
DR	=	Maximum annual discount rate applied to the market of this kind of assets.
tm	=	Estimated time in number of months to reach the ER'.

For the determination of variables of this procedure it is necessary to adjudicate a DR value that is equivalent to the

maximum value assigned by a potential purchaser of this kind of assets, in view of the fact that the objective is to fix a minimum price of reference; this market value for DR is selected from the information gathered by the firm through more than 80 placement processes of loan assets in the Mexican market, taking into consideration the market conditions that may prevail at the time LAMM is applied

The value of DR represents the present value of the annual return expected by a potential investor applied as annual prospective discount on the ER value.

In accordance with the conditions that may prevail at the market upon the placement of the loan assets, we may affirm that the value of annual discount rate (DR) is inversely proportional to the level of the market demand and the concentration of competence among the potential investors, during the process.

The variable defined as t_m or estimated period of time in number of months to reach the ER is determined from the information incorporated to carry out the estimate of PP within the Probit model already described and is an integral part of the activities of due diligence that take place during the gathering of information process in the portfolio analysis and it is estimated as the maximum number of months where the probability of $y = 1$ to reach ER' is considered.

REFERENCES AND SOURCES

Altman E. (1968), "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy", *Journal of Finance*, September, 1968, New York.

Altman E., Haldeman R and Narayanan P. (1977), "ZETA Analysis: A new model to identify bankruptcy risk of Corporations",

Journal of Banking and Finance, June 1, 1977.

Altman, Heine, Hartzell & Peck (1995), "Emerging Markets Corporate Bonds: A Scoring System", Salomon Brothers Inc., New York.

Altman E. & Saunders A. (1998), "Credit risk measurement: Developments over the last twenty years", *Journal of Banking & Finance*. p.p. 1,721-1,742.

Arias-Corres, J.C. (2006), "Model Averaging and Value-at-Risk Based on Multi Asset Volatility Models for Risk Management", *Proceedings of the EBS*. Cambridge, UK. November 2006.

Basle Committee on Banking Supervision (1999), "Credit Risk Modeling: current practices and applications", Basle Switzerland, April 1999.

Basle Committee on Banking Supervision (2000), "Principles for the management of credit risk", Basle Switzerland, September 2000.

Beaver, W. (1967), "Financial ratios as predictors of failure", *Empirical Research in Accounting*. (1966), "Selected studies", supplement to the *Journal of Accounting Research*, January, 1966.

Bolem Adriaan M. and Russel Freeman (2006), "The Treatment of Nonperforming Loans: Clarification and Elaboration of Issues Raised by December 2006 Meeting of The Advisory Expert Group of the Intersecretariat Working Group on National Accounts", *International Monetary Fund*, United Nations Statistics Division, December 2006.

- Cortavarria, L., Dziobek Claudia, Kanaya Akihiro and Inwon Song (2000), "Loan review, Provisioning, and Macroeconomic Linkages", International Monetary Fund, Working Paper, December 2000.
- Crosbie (1997), "Modeling Default Risk". KMV Corporation, San Francisco.
- Deakin, E.B. (1972), "A Discriminant Analysis of Predictors of Business Failure", Journal of Accounting Research.
- Frydman H., Edward I. Altman & Kao Duen-Li (1985), "Introducing Recursive Partitioning for Fiscal Classification: The Case of Financial Distress", The Journal of Finance, vol. XL, No. 1, March 1985.
- Greene W. (1993), *Econometric Analysis*, 2th Edition, Mc. Millan Publishing Co.
- Heine, M. (2000), "Predicting financial distress of companies: revisiting the Z-Score and ZETA Models", New York, June 2000.
- Jorion, Phillip (1997), *Value-at-Risk*, The McGraw – Hill Companies.
- J.P. Morgan, G. Gupton, C. Finger & M. Bathia (1997), "Credit Metrics – Technical Document", J.P. Morgan and Co., New York, April 2, 1997.
- Kealhofer (1993), "Portfolio Management of Default Risk", KMV Corporation, San Francisco.
- Kealhofer, Kwok, Weng (1998), "Uses and Abuses of Bond Default Rates", KMV Corporation, San Francisco.
- Mandala G.S. (1983) "Limited Dependent and Qualitative Variables in Econometrics", Econometric Society, Monographs number 3, Cambridge, University.
- Market Intelligence Service (2004), "Estimated Return for Secured Nonperforming Asset Portfolios Model", Sayco, Mexico.
- Market Intelligence Service (2004), "Estimated Return for Consumer Loan Portfolios Model", Sayco, Mexico.
- Market intelligence Service (2005), "Estimated Return for Secured Performing Asset Portfolios Model", Sayco, Mexico.
- Narayan U. (1972) "Elements of Applied Stochastic Processes", John Willie & Sons Inc.
- Nishimura, Kiyohiko G. and others (2001), "The Disposal of Non-Performing Loans and Its potential influence", Director General for Economic Assessment and policy Analysis, Cabinet Office, Government of Japan.
- Santos Téllez, Carla, 1999, "Riesgo de Crédito en México: El enfoque de portafolios" "Risk of Credit in Mexico: The focus on portfolios", XV Premio Nacional de Investigación Financiera del Instituto Mexicano de Ejecutivos de Finanzas (XV National Award on Financial Research granted by the Instituto Mexicano de Ejecutivos de Finanzas), Thesis ITAM, México.
- Sellers, Vasicek & Levinson (2000), "The KMV EDF Credit Measure and Probabilities of Default", KMV Corporation, San Francisco.

Tobin J. (1958), "Estimate of Relationships for Limited Dependent Variables", *Econometrica*.

Vasicek (1984), "Credit Valuation" *KMV*

Corporation, San Francisco.

Wilson Tomas (1997), "Portfolio Credit Risk", *Risk Primer*, September, vol. 10, number 9.